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Are elementary particles point-like objects?

P. Brovetto[†], V. Maxia[†] and M. Salis[†‡] [1]
[†]On leave from Istituto Nazionale di Fisica della Materia - Cagliari, Italy
[‡]Dipartimento di Fisica - Università di Cagliari, Italy

Abstract - Simple arguments based on uncertainty principle and Dirac's equation are examined which show that electron behaves either as a point-like charge or as an extended distribution according as high- or low-energy experiments are considered.

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Nowadays many physicists agree that elementary particles are point-like objects. This opinion is based mostly on results of late experiments on high-energy electron-positron collisions. These experiments show that the electron size is less than 10^{-16} cm, that is, more than three orders of magnitude smaller than the classical electron radius. In these experiments, the collision time τ, as determined in the centre of mass of two opposite-charge electrons moving with velocities $+v$ and $-v$, can be roughly identified with the ratio between impact parameter b and electron velocities, that is, $\tau \simeq b/v$ [1]. The impact parameter in direction orthogonal to velocities is assumed equal to the electron size, that is, $b_\perp = 10^{-16}$ cm. But $b_{\shortparallel} = b_\perp \left(1-\beta^2\right)^{1/2}$, owing to Lorentz contraction in direction parallel to velocities. With electrons of energy 45 GeV, we get $\left(1-\beta^2\right)^{1/2} = m_e c^2/\left(45\text{ GeV}\right) = 1.1\cdot 10^{-5}$ which, by letting $v = c$, leads to $\tau \simeq b_{\shortparallel}/c = 10^{-16}\left(1-\beta^2\right)^{1/2}/c = 3.7\cdot 10^{-32}$ s. This exceedingly small value of τ shows that the electron-positron collision dealt with is indeed a very fast process. The situation, of course, is quite different in low-energy experiments, as for instance in atomic spectroscopy which involves long-living electron states. In general, a high energy entails a "fast" process while a low energy entails a "slow" process.

In order to decide whether electron shows a point-like nature also in low-energy experiments, it is convenient to write the energy-time uncertainty principle in the dimensionless form

$$\frac{\delta w}{m_e c^2}\cdot\frac{\delta t}{T_Z} \simeq 1, \tag{1}$$

m_e standing for the electron rest mass, and

$$T_Z = \frac{h}{m_e c^2} = 8.1\cdot 10^{-21}\text{s} \tag{2}$$

for twice the so-called Zitterbewegung period. As for the meaning of this quantity, let us briefly recall some issues of Dirac's equation concerning electron

[1]Corresponding Author e-mail: masalis@unica.it

velocity [2, 3, 4, 5]. According to this equation, expected electron velocity on x-axis is [3]

$$\left\langle \dot{x}\left(t\right)\right\rangle =\iiint\sum_{k=1}^{k=4}\psi_{k}^{*}\left(\overrightarrow{r},t\right)\left(-c\alpha_{1}\right)\psi_{k}\left(\overrightarrow{r},t\right)d^{3}\ \overrightarrow{r}, \tag{3}$$

where ψ_k are the spinor components. Fourier's expansion allows us to write

$$\psi_{k}\left(\overrightarrow{r},t\right)=\iiint\left\{a_{k}\left(\overrightarrow{p}\right)\exp\left[\left(2\pi i/h\right)\left(Wt-\overrightarrow{p}\cdot\overrightarrow{r}\right)\right]+\right.$$

$$\left.+b_{k}\left(\overrightarrow{p}\right)\exp\left[\left(2\pi i/h\right)\left(-Wt-\overrightarrow{p}\cdot\overrightarrow{r}\right)\right]\right\}d^{3}\ \overrightarrow{p} \tag{4}$$

where W means the electron energy and a_k, b_k the amplitudes of positive and negative energy states, respectively. The latter are essential because it was shown that without their contribution electrons cannot originate Thomson scattering of light. This scattering appears as a sort of resonance of the quantum jump of energy $2m_ec^2$ between positive and negative energy states [6]. Substituting eq. (4) into eq.(3), and integrating over $\overrightarrow{r}$ originates terms $-ca_k^*\alpha_1 a_k$ and $-cb_k^*\alpha_1 b_k$ which lead to the Ehrenfest theorem as in non-relativistic quantum mechanics. Terms $-ca_k^*\alpha_1 b_k\exp\left(-4\pi iWt/h\right)$ + H.C. are also found which couple positive with negative energy states. They represent oscillations of velocity of angular frequency $4\pi W/h$. Since we are dealing with low energy electrons, we assume that W barely exceeds m_ec^2. So, by putting $W=m_ec^2$, we have from eq. (2) $W/h=1/T_Z$. Consequently, with obvious meaning of labels E and Z, the expected velocity can be written as

$$\left\langle \dot{x}\left(t\right)\right\rangle =\left\langle \dot{x}\left(t\right)\right\rangle _{E}+\left\langle \dot{x}\left(t\right)\right\rangle _{Z}. \tag{5}$$

Component $\left\langle \dot{x}\left(t\right)\right\rangle _{E}$ is not at issue. As for component $\left\langle \dot{x}\left(t\right)\right\rangle _{Z}$, by performing some transformatios, we get

$$\left\langle \dot{x}\left(t\right)\right\rangle _{Z}=c\iiint A_{1}\cos\left(4\pi\frac{t}{T_{Z}}+\varphi_{1}\right)d^{3}\ \overrightarrow{p}, \tag{6}$$

where amplitude A_1 and phase φ_1 are real quantites, depending on $\overrightarrow{p}$, which can be evaluated in function of Fourier's amplitudes a_k and b_k. Integration over time, yields in turn

$$\left\langle x\left(t\right)\right\rangle _{Z}=\frac{\lambda_{C}}{4\pi}\iiint A_{1}\sin\left(4\pi\frac{t}{T_{Z}}+\varphi_{1}\right)d^{3}\ \overrightarrow{p}, \tag{7}$$

$\lambda_C = cT_Z = h/m_ec$ standing for Compton's wavelength. Taking also into account components $\left\langle y\left(t\right)\right\rangle _{Z}$ and $\left\langle z\left(t\right)\right\rangle _{Z}$, it follows that each element $dp_xdp_ydp_z$ of momentum space is associated with a closed trajectory formed by electron oscillations on x, y, z axes. Adding up these trajectories, a complex dynamic substructure is originated in space around the electron centre of mass [7]. So

far an exhaustive treatment on this matter has not been implemented, due perhaps to the difficulty of rightly define the low-energy spinor ψ_k in eq. (3).

Coming back to eq. (1), we point out that in low-energy experiments we have in general $\delta w/m_e c^2 \ll 1$. Indeed, in atomic spectroscopy energies are determined with accuracies so good that the Rydberg constant, $R_H = 13.5056981$ eV, is known with a precision of seven decimal digits. This entails $\delta w < 10^{-7}$ eV and $\delta w/m_e c^2 < 2 \cdot 10^{-13}$. Accordingly, we obtain from eq. (1) $\delta t/T_Z > 5 \cdot 10^{12}$. It follows that oscillations considered in eqs. (7) and (6) are not observable since indetermination in time largely exceeds oscillation period. Consequently, time must be eliminated in previous equations so that oscillations are changed into a probability density distributed on the electron trajectories [2]. In our opinion, this argument, although only qualitative, is sufficient to conclude that in low-energy experiments electron behaves as an extended charge of definite size.

This outcome is not surprising because in quantum physics electron cannot be regarded as an "absolute" entity. Electron, on the contrary, is merely an "observable" object. It follows that in actual experiments different sides of electron can be probed, depending on the experiment considered. So, electron is point-like in fast collisions at high energy ($\tau \ll T_Z$) but is extended in low-energy atomic spectroscopy. Since all elementary particles are fermions represented by Dirac's equation, this results should be applied to all particles barring neutrinos. Indeed, their yet unknown vanishingly small mass prevents eq. (2) to be applied. We point out, finally, that the rest-mass of the extended electron might be ascribed to its own electromagnetic field without run into divergent quantities as with the point-like option. Some conjectures about this matter are reported in Ref. [8]

References

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[2] This is like what occurs when dealing with a classic oscillator $x = Asin\,(2\pi t/T)$. Probability that the oscillating particle is found in space between x and $x + dx$ is $dP = 2dt/T$, dt standing for the time required to cross dx. We have thus $dP/dx = 2/\left(\dot{x}T\right)$ which allows us to write $dP/dx = 1/\left(\pi\sqrt{A^2 - x^2}\right)$ (see Ref. [8]). A simple way to grasp the argument at issue is as follows. In Bohr-Sommerfeld mechanics atomic electrons move on closed trajectories. In quantum mechanics these trajectories are replaced by time-independent clouds of probability just as a consequence of energy-time uncertainty principle.